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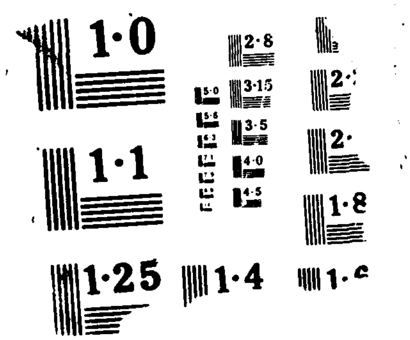
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Nonlinear Dynamics of Coupled Oscillator Arrays

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The phase-locked dynamics of large oscillator arrays is currently of interest because of possible microwave directed energy applications. Straight-forward integration of the coupled dynamical equations for such arrays is computationally costly for the associated multidimensional parameter space, long integration times, various initial conditions and system configurations. Finite difference analogues of the nonlinear differential equations can reproduce their complex dynamical behavior with a 2 to 3 order-of-magnitude improvement in computational time.

Here, the applicability of the finite difference technique is demonstrated by solutions of the dynamical equations for 2 coupled oscillators and rings of larger numbers. Parameter studies for these configurations suggest the values of coupler length and coupling strength required to provide robust phase-locked operation. The finite difference technique can be extended to model large oscillator arrays with other coupling geometries, amplifier arrays, and additional physical phenomena.

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CONTENTS

NONLINEAR DYNAMICS OF COUPLED OSCILLATOR ARRAYS

I. INTRODUCTION

Computers have made possible the solution of fundamental nonlinear dynamical equations in many scientific disciplines.[1] Along with the numerical work, sophisticated group-theoretical techniques have been employed to study similarities of behavior in large classes of dynamical equations.[2] One highly studied class, driven nonlinear oscillators, includes the equations of coupled oscillators that model sources designed to meet high-power, directed microwave energy objectives. Research in this area of nonlinear dynamics has demonstrated the following:

* Simple difference equation analogues can faithfully reproduce the rich spectrum of dynamical behavior predicted by the nonlinear differential equations.

* The dynamical behavior (following decay of initial transients) depends sensitively on key constants called control parameters.

* Variations in the control parameters can produce abrupt (and frequently counter-intuitive) changes in the oscillator's long-term behavior between free-running, phase-locked, quasiperiodic-subharmonic, and chaotic states.

* The changes from well-ordered to chaotic behavior occur via a few paths of universal character; they are similar for a wide variety of force laws obeying a few general requirements.

These characteristics allow one to employ difference equation analogues to study coupled microwave oscillator arrays.

A general objective of such nonlinear studies is to determine which portions of the control parameter space are associated with the different dynamical behaviors. For the work of this report, this objective takes the following form: What values of the experimental parameters (cavity Q, coupler length, etc.) and what system geometries are required to insure robust phase-locked behavior in coupled oscillator arrays relevant to directed microwave energy objectives?

The major benefit of the difference equation approach over the direct numerical solution of the differential equations is a two to three order-of-magnitude increase in computational speed. Though not required for small oscillator arrays, the increase in speed may be crucial for the economic study of large arrays with the associated multi-dimensional parameter space, long integration times, various initial conditions and configurations. (For

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example, one configuration of 1000 coupled oscillators, integrated for 100 periods with 100 steps/period, and 100 combinations of 2 parameters requires 10^9 differential equation time steps).

The utility of the difference equation approach is illustrated in the next section, where the method is applied to the two-coupled-oscillator experiment of Benford and Woo[3] and compared to their analysis. In Sec. III, the problem is generalized to a ring of oscillators with nearest-neighbor interactions. Calculations with as many as 16 oscillators are presented and results are discussed. In Sec. IV, issues associated with large arrays and techniques to include additional experimental phenomena, such as noise and pulsed-power variations are briefly discussed.

II. Two Coupled Oscillators

The starting point for this analysis is the differential equation formulation of Benford and Woo.

$$\frac{d\phi_1(t)}{dt} = \omega_{o1}\left\{1 - \frac{\rho_1}{Q_1}\sin[\phi_1(t) - \phi_2(t - \tau_p)]\right\} \tag{1}$$

$$\frac{d\phi_2(t)}{dt} = \omega_{o2}\left\{1 - \frac{\rho_2}{Q_2}\sin[\phi_2(t) - \phi_1(t - \tau_p)]\right\} \tag{2}$$

where $\phi_{1,2}$ are the instantaneous phases in oscillators 1 and 2, $\omega_{o1,o2}$ are their free-running angular frequencies, $Q_{1,2}$ are the cavity quality factors, $\rho_{1,2}$ are related to the ratios of injected and output powers (in the range of 0.1 to 1), and τ_p is the phase transit time through the structure coupling the two oscillators.

The authors have shown that steady, phased-locked behavior

$$\frac{d\phi_1}{dt} = \frac{d\phi_2}{dt} = \omega = \text{constant}.$$

obeys

$$\sin\Delta\phi = \frac{\varepsilon}{k\cos\phi_c} \tag{3}$$

where $\Delta\phi = \phi_2 - \phi_1$, $\varepsilon = \Delta\omega/2\omega_o$, $\Delta\omega = \omega_{o2} - \omega_{o1}$, $\omega_o = (\omega_{q1} + \omega_{o2})/2 >> \Delta\omega$, $k = \rho/Q$ is the same for the two oscillators, and $\phi_c = \omega\tau_p$. From Eqn.(3), phase locking is possible when

$$k|\cos\phi_c| \geq \varepsilon \tag{4}$$

If phase-locking does occur

$$\omega = \omega_o(1 - k\cos\Delta\phi\sin\phi_c) \tag{5}$$

Note that Eqn.(4) is a necessary but insufficient condition for phase locking. The dynamical equations must be solved subject to particular initial conditions to determine if locking does occur for particular values of the control parameters ϕ_c, ε and k.

The straight-forward way to solve Eqns.(1) and (2) is to choose a small time increment $\Delta t << 2\pi/\omega_o$ and iterate

$$\phi_1{}^{n+1} = \phi_1{}^n + \omega_o\Delta\tau(1-\varepsilon)[1 - k\sin(\phi_1{}^n - \phi_2{}^{n-q})] \tag{6}$$

$$\phi_2{}^{n+1} = \phi_2{}^n + \omega_o\Delta\tau(1+\varepsilon)[1 - k\sin(\phi_2{}^n - \phi_1{}^{n-q})] \tag{7}$$

where $\phi^{n+1}_{1,2} = \phi_{1,2}(n\Delta t)$ and $q\Delta t = \tau_p$.

Nonlinear dynamics research has shown that the long term evolutions of such equations preserve their character even when $\Delta t > 2\pi/\omega_o$. For the problem at hand, the natural choice is $\Delta t = \tau_p$ leading to the 2-dimensional iterative map

$$\phi_1{}^{n+1} = \phi_1{}^n + 2\pi W(1 - \varepsilon)[1 - k\sin(\phi_1{}^n - \phi_2{}^{n-1})] \tag{8}$$

$$\phi_2{}^{n+1} = \phi_2{}^n + 2\pi W(1 + \varepsilon)[1 - k\sin(\phi_2{}^n - \phi_1{}^{n-1})] \tag{9}$$

where $W = \omega_o\tau_p/2\pi$ is close to the number of wavelengths in the coupler $\phi_c/2\pi$. The equivalent 1-dimensional map, the sine circle map, is the subject of much analysis.[4-6]

When phase-locked, $\Delta\phi^n = \phi_2{}^n - \phi_1{}^n$ is constant, and the difference equations yield locking conditions identical to Eqns. (3)-(5). One can then be confident that they emulate the differential equations even though the iteration time is several periods of the oscillation rather than a small fraction of it. It is now shown that the evolution of $\Delta\phi$ depends sensitively on the control parameters W and k.

To illustrate the range of long-term dynamical behavior accessible to a pair of coupled oscillators, consider solutions of Eqns.(8) and (9) for $W = 2$, $\varepsilon = .01$, and various values of k (Fig. 1). In the left-hand frames, an initial phase difference is chosen, and the first 100 iterations of Eqns.(8) and (9) are plotted. The right-hand frames plot the iterations of 8 initial phase differences spanning 2π radians.

When $k = .0075$, the coupling is too weak for locking and $\Delta\phi$ changes monotonically for the distorted free-running oscillators. This value of k does not satisfy Eqn.(4). When k exceeds .01, all initial conditions eventually lock to the phase difference predicted by Eqn.(3). At about $k = .09$, phase locking for all initial conditions is replaced by a stable 3-cycle in which the appearance of a nonlinear subharmonic instability leads to oscillations between three values of $\Delta\phi$. At about $k = .11$, the three phases broaden into nearly aperiodic bands. Above $k = .14$, the bands merge and the evolution looks chaotic. Note that the larger k value cases satisfy Eqn.(4) but are not phase-locked.

Data of the sort illustrated in Fig. 1 are summarized in Fig. 2, where 100 successive iterations starting with $n = 200$ are plotted for 16 initial conditions at each of 400 k values. In this way, phase-locking for the ensemble of initial conditions at fixed k shows as a point, the "trifurcation" as a vertical array of 3 points, and chaotic evolution as a vertical line. Though the first 200 iterations of each computer experiment must be calculated, they are not plotted so that only the long term evolution, free of initial transients, is displayed.

Figure 3 summerizes the dynamic evolution for fixed k and varying W using the approach described for Fig. 2. In the region where phase-locking occurs, the plots are equivalent to the analytic phase difference vs transit time plots of Benford and Woo. The plots show a hashy fill from top to bottom in the regime where Eqn.(4) cannot be satisfied. A new feature of the dynamic solutions is the inability to phase-lock at large delays when coupling is strong. These, and other solutions, demonstrate that the onset of subharmonic instabilities produces an upper limit to coupling strength and coupler length determined from

$$Wk < .16 \tag{10}$$

for phase-locking of two oscillators. The equivalent condition for large oscillator arrays may represent a stringent limit to their design.

III. <u>Extension to Larger Arrays</u>

The simplest way to extend the analysis to larger oscillator arrays is to consider a ring with nearest neighbor coupling. This geometry minimizes the number of couplings to each oscillator, and avoids the treatment of free bounderies. The differential equation governing the i^{th} oscillator in a ring of N is

$$\frac{d\phi_i}{dt} = \omega_{oi}\left\{1 - k\sin[\phi_i(t) - \phi_{i-1}(t-\tau_p)] - k\sin[\phi_i(t) - \phi_{i+1}(t-\tau_p)]\right\} \tag{11}$$

where all the k factors are assumed identical. It is also assumed that all cavity free-running frequencies are

the same so that locked phase differences $\Delta\phi$ between neighboring oscillators will be equal. Adding all the phase differences around a phase-locked ring leads to the requirement

$$N\Delta\phi = 2\pi m \ ; \ m = 0, \pm 1, \pm 2 \ldots \qquad (12)$$

For small arrays, operation with m = 0 is probably most effective for coherent power amplification. In any case, uncontrolled shot-to-shot variations in phase difference are not desirable, so that robust phase-locking at a predetermined value of m is required.

The difference equation analogue of Eqn.(11) is used to study which phased-locked m occurs for given initial conditions and control paramter values.

$$\phi_i^{n+1} = \phi_i^n + 2\pi W \left\{ 1 - k[\sin(\phi_i^n - \phi_{i-1}^{n-1}) + \sin(\phi_i^n - \phi_{i+1}^{n-1})] \right\} \qquad (13)$$

Here, i = 1 to N, $\phi_{N+1} = \phi_1$, and $\phi_o = \phi_N$, and the control parameters have their previous definition. A large number of initial phases are chosen randomly and their long term evolution is examined. Figure 4 shows the first 100 iterations of two evolutions for N = 6, k = .04, and W = 1. Each oscillator's history is labeled according to its position in the ring. For about 90% of the initial conditions, the evolution resembles Fig. 4A, rapid phase locking with m = 0. About 10% of the time, the initial conditions relax to an m = $\pm$1 configuration as shown in Fig. 4B.

Computer runs with other N values and integral W values show that m = 0 occurs exclusively for $N \leq 4$, that the incidences of m = $\pm$1 become more frequent as N increases and that m = $\pm$2 appears occasionally for N in the teens. The phase-locking condition equivalent to Eqn.(10) for a ring of oscillators is

$$Wk < .08 \qquad (14)$$

for all values of N studied. This is reasonable since the two couplers for each oscillator have a combined coupling strength of 2k. It is conjectured that the condition for a regular array of oscillators with η couplings to each oscillator is $Wk < .16/\eta$.

Though not intuitive, nonlinear analysis indicates that improved reproducibility and robustness of lock-in may result from certain noninteger W values.[4-6] This improvement is demonstrated in Fig. 5, constructed in the manner of Fig. 2 with N = 16. Each of the $\Delta_i\phi = \phi_i - \phi_{i-1}$ values is plotted each time step for 10 randomly selected initial conditions and a given k value. When W = 10, lock in occurs in accordance with Eqn.(14) at m = 0, $\pm$1 and occassionally m = +2. When W = 8.8, all initial conditions collapse to m = 0 phase-locking and the critical Wk

product is increased. Careful design of the couplers may therefore improve reproducibility in the experiment. Initial calculations indicate that small variations in ω_{o_i} between oscillators may also improve robustness of phase locking at $m = 0$.

IV. Areas of Continued Research

The finite difference techniques described above can be extended to large arrays with different coupling geometries to determine which configurations and experimental parameters lead to rapid and robust phase-locking. One possible limitation to performance in large arrays is described in an analysis of phase organization in a large linear array of driven Hamiltonian oscillators.[7] There, the system is shown to relax on two time scales: a fast-time in which local regions phase-lock and a much longer time (10^4 iterations) in which these local regions interact and merge. One can well imagine local groups of phase-locked microwave oscillators in a large array with chaotic oscillations at group boundaries for the duration of the power pulse. It is presently unclear if more clever grouping and coupling of oscillators can alleviate this difficulty. One possible solution may involve seed amplifiers to keep local oscillator groups in step.

Additional physical phenomena can be included in the finite difference analysis. For example, noise tolerance of frequency locked dynamics has been studied[8] by adding a white noise term to the sine circle map. A similar procedure can be employed with oscillator arrays to study the effects of experimental noise or statistical effects due to unresolved degrees of freedom. Also, time variations in pulsed power can be studied by applying slow time variations to the control parameters.

A major improvement in the analysis would be to describe each oscillator by two difference equations involving the phase and field amplitude. Descretized versions of the pendulumm-like equations derived by Fliflet[9] or the generalized VanderPol oscillators researched by Walsh, Johnston and Davidson[10] could be employed. Then, mixed amplifier-oscillator configurations could be studied as well as operation with time-varying pulsed power.

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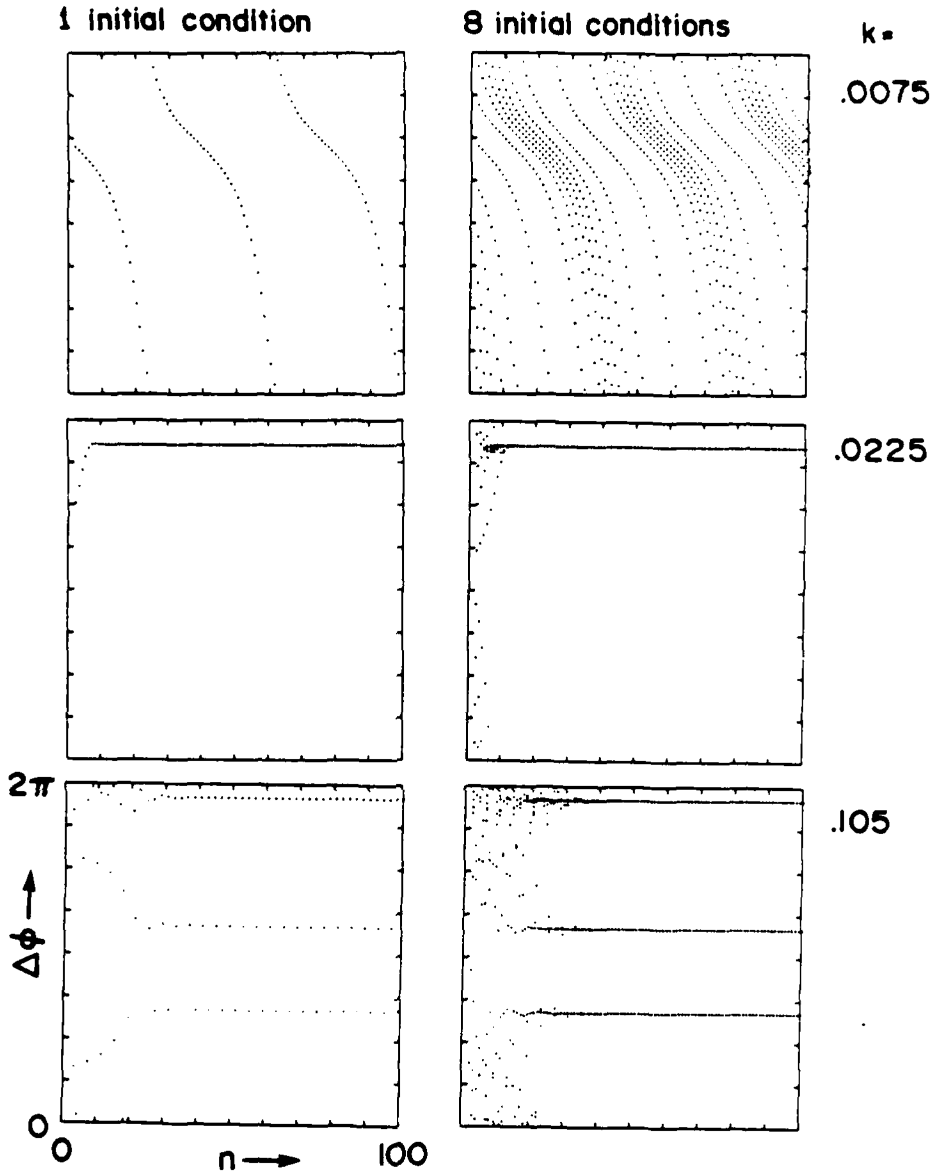

Fig. 1 — Evolution of phase difference between 2 coupled oscillators for $W = 2$, $\epsilon = .01$ and 5 values of k. The left hand frames plot the first 100 iterations of 1 initial condition. Those on the right superimpose 8 initial conditions.

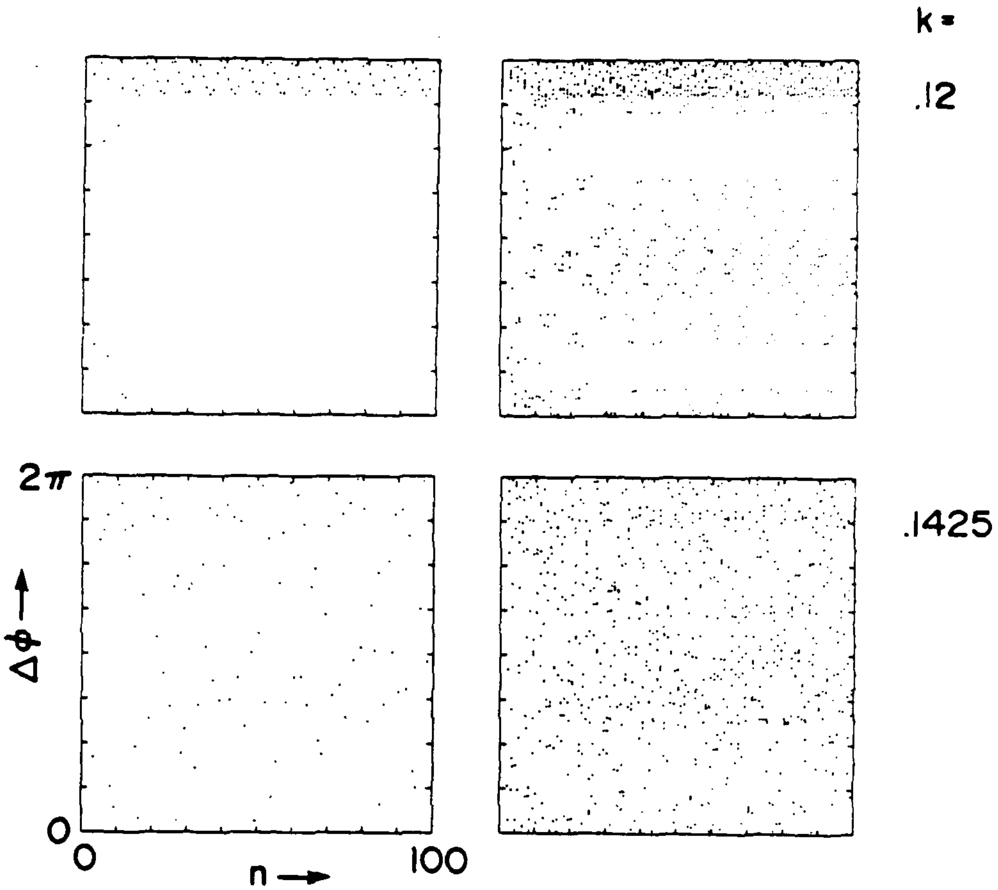

Fig. 1 — (Continued) Evolution of phase difference between 2 coupled oscillators for $W = 2$, $\epsilon = .01$ and 5 values of k. The left hand frames plot the first 100 iterations of 1 initial condition. Those on the right superimpose 8 initial conditions.

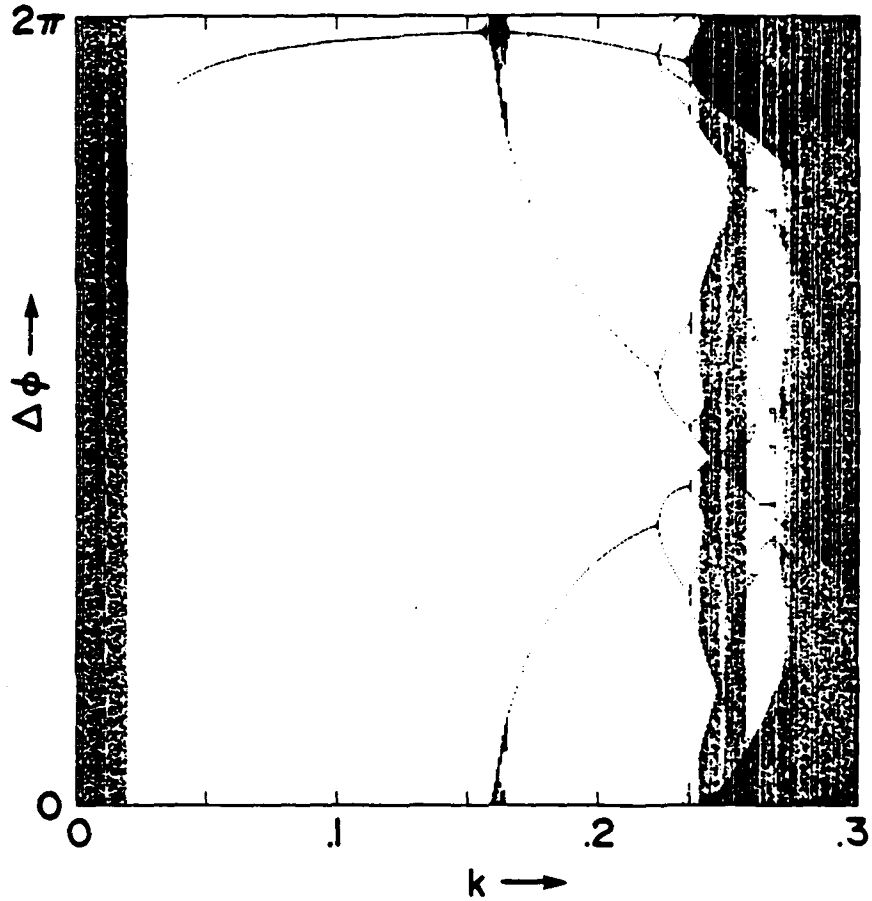

Fig. 2 — Long term evolution of phase difference vs. k for W = 2 and ϵ = .01.

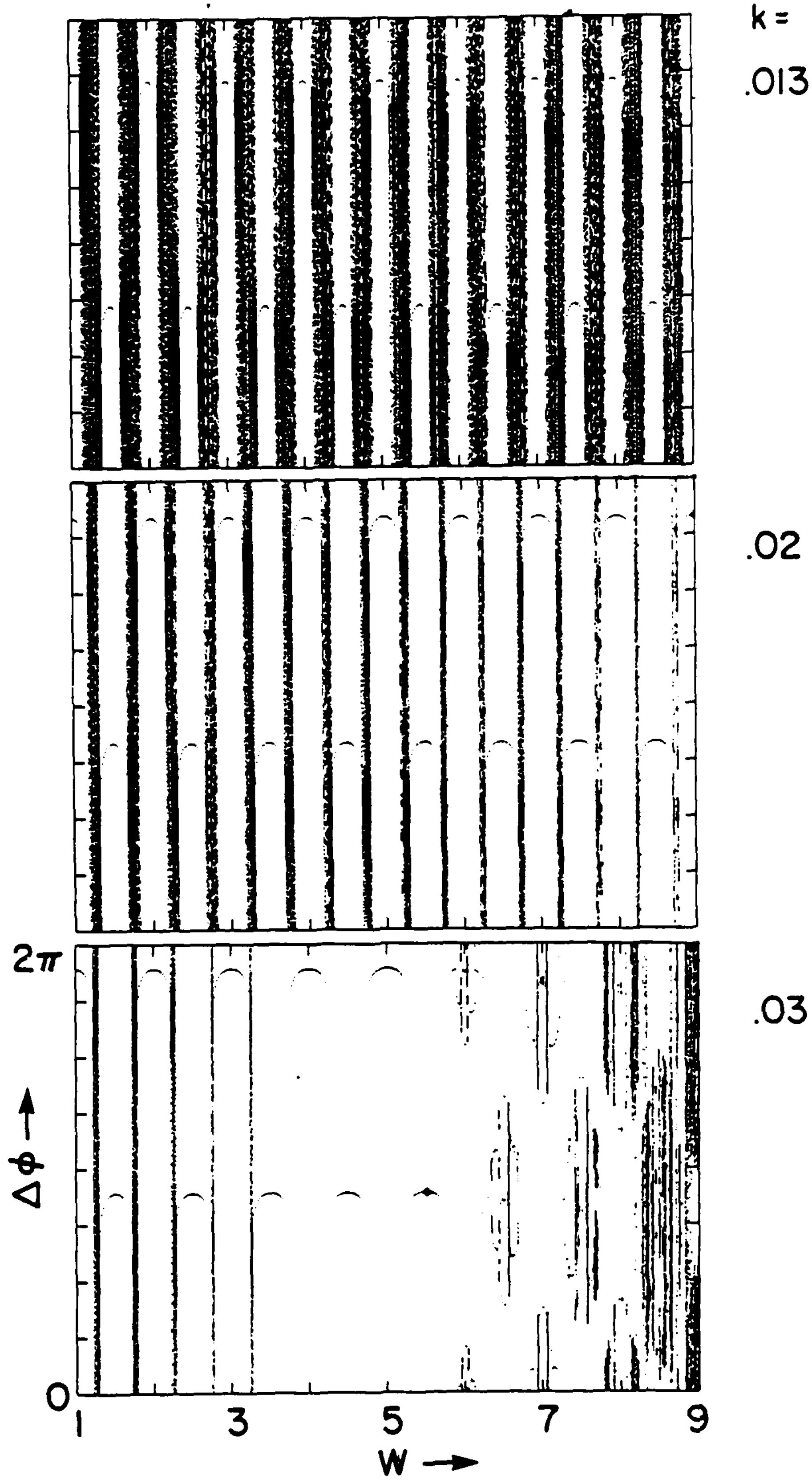

Fig. 3 — Phase difference vs. W for ϵ = .01 and 3 values of k.

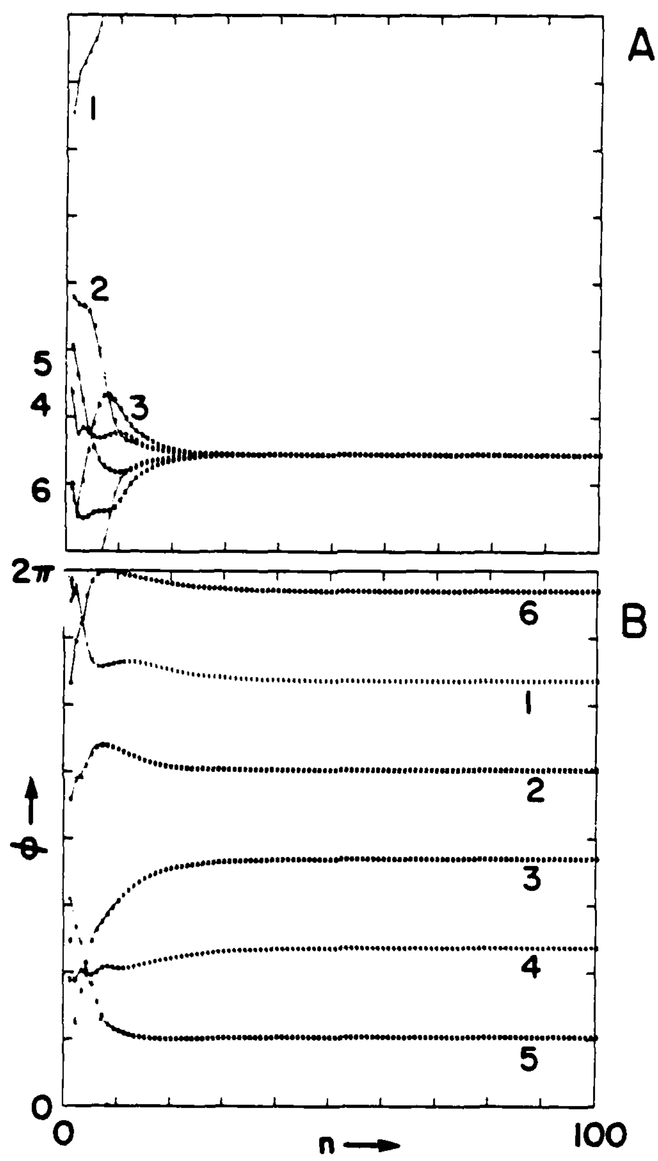

Fig. 4 — Phase evolution of a 6 oscillator ring for W = 1, k = .04 and 2 random initial conditions.

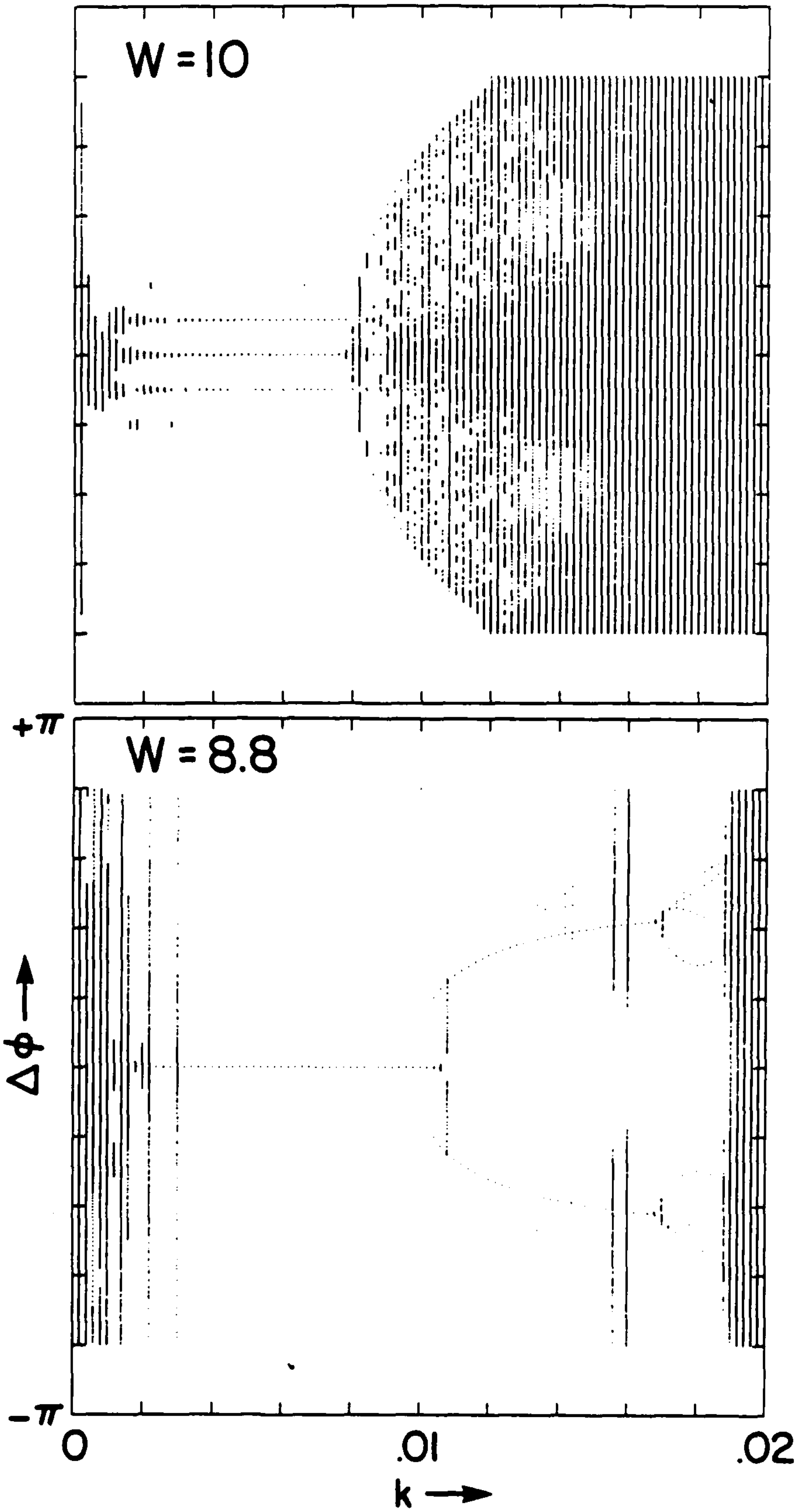

Fig. 5 — Phase differences vs. k for a ring of 16 oscillators and 2 values of W.

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